CHAPTER 12 -- PROPERTIES OF MATTER & STATATIC ELECTRIC FORCES & FIELDS

QUESTION & PROBLEM SOLUTIONS

12.1) The mass of an electron is $9.1\text{x}10^{-31}$ kg and its charge is $1.6\text{x}10^{-19}$ coulombs. If two electrons are separated by 1 meter, each will exert an electrical force and a gravitational force on one another. How do those forces compare?

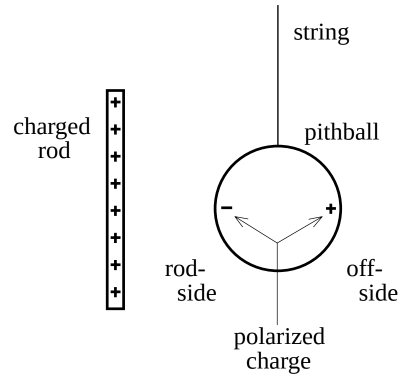
Solution: The magnitude of the electrical force between any two *point charges* is called a *Coulomb force*. It is numerically equal to kq_1q_2/r^2, where k is a constant equal to $9\text{x}10^9\ nt{\cdot}m^2/C^2$ (it's normally written as $1/(4\pi\varepsilon_o)$, where ε_o is, itself, a constant called the permittivity of free space), the q terms are the magnitudes of the charges (in the MKS system, the units of charge are *coulombs*), and r is the distance in meters between the point charges). For two electrons, this force becomes $(9\text{x}10^9\ nt{\cdot}m^2/C^2)(1.6\text{x}10^{-19}\ C)^2/(1\ m)^2 = 2.3\text{x}10^{-28}\ nts$. The magnitude of the gravitational force between any two masses is Gm_1m_2/r^2, where G is the Universal Gravitational Constant equal to $6.67\text{x}10^{-11}\ nt{\cdot}m^2/kg^2$, the m's are masses, and r is the distance between the centers of mass of the two bodies. For the two electrons, this equals $(6.67\text{x}10^{-11}\ nt{\cdot}m^2/kg^2)(9.1\text{x}10^{-31}\ kg)^2/(1\ m)^2 = 5.52\text{x}10^{-71}\ nts$. The ratio of these two forces is $F_e/F_g = 4.17\text{x}10^{42}$. That is, the electrical force is 10^{42} times larger.

12.2) A light, small, styrofoam ball (this is called a *pith ball*) is painted with a metallic paint and attached to a string that hangs freely in mid-air.

a.) What will the pithball do when a positively charged rod is brought close to it (the two don't touch)?

Solution: The bare bones information you need to understand this is as follows: 1.) Structures are deemed *metallic* if their atomic bonding allows their valence electrons (i.e., outer shell electrons) to roam freely throughout the material. Protons, on the other hand, are fixed within the nucleus of the atoms *and cannot move around*. 2.) Without the presence of an outside charge like that on the rod, the natural repulsion between the negative electrons motivates them to distribute evenly over the outside of a spherical, metallic surface. 3.) When a positive rod approaches such a surface, the electrons in the metal are attracted to and migrate toward the rod (negative charge is attracted to positive charge). 4.) The flow of electrons toward the rod-side of the sphere ceases when the natural repulsion between electrons makes it impossible for more electrons to join the crowd. 5.) At that point, the rod-side of the sphere is electrically negative. 6.) With positively charged protons fixed in the atomic lattice, and with electrons having moved to the rod-side of the sphere, the off-side is left with a net positive charge. 7.) In other words, the sphere has been electrically polarized.

With all of this in mind, what does the pithball do when the rod approaches? The electron shuffle (sounds like a line dance) produces a relatively large negative charge on the rod-side of the sphere which is *closer* to the rod than is an equally large *positive* charge left on the off-side of the sphere. The net effect is that the attraction between the rod and negative charge on the pithball will be greater than the repulsion between the rod and the positive charge on the off-side of the pithball, and the net force will motivate the pithball to move toward the rod. In short, the pithball will swing toward the rod.



b.) How would the results of *Part a* have changed if the rod had been negatively charged?

Solution: The only difference between the two cases is that the electrons on the rod-side of the pithball would migrate away from the negatively charged rod leaving the rod-side electrically positive and the off-side electrically negative. The positive side would be closer to the rod than the negative side, so once again there will be a net attraction and the pithball would swing up toward the rod.

c.) How would the results of *Part a* have changed if the pithball had not been coated with a metallic paint but, instead, was simply styrofoam?

Solution: The temptation is to assume that because there is no metallic bonding here, the electrons in the pithball will not be able to move and, hence, nothing will happen. It turns out that that isn't the case. Indeed, electrons in a covalently bonded material cannot migrate throughout the material as they can with metallic bonding, but they do move *within the atom*. Under normal circumstances (i.e., without a lot of outside charge around), the geometric center of "orbiting" electrons is at the center of the atom. That is, on average, the electrically negative part of an atom is centered at the same place where the electrically positive part of the atom (i.e., the proton) resides--at the center of the nucleus. When a positively charged rod comes close, the electrons in each atom of the structure spend more time on the rod-side of their respective atoms. In other words, the average position of the negative charge in the individual atoms no longer "covers" the positive charge--there is a very slight polarization. This is, indeed, VERY SLIGHT (after all, it is all happening inside atoms--structures that are only 10^{-10} meters across), nevertheless the shift creates a disparity between the electron's attraction to the rod and the proton's repulsion to the rod. The net effect is that the pithball will, again, feel a net force and will swing toward the rod.

d.) The rod and pithball in *Part a* touch. What are the consequences for the pithball?

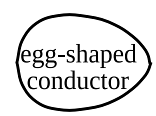
Solution: When the two touch, electrons on the pithball will transfer to the rod. This new negative charge density will be large where the contact occurs and nonexistent at other places (the rod is covalently bonded so electrons can't roam about through it as would be the case if the rod had been metallic). Having lost electrons, the pithball now has a net charge that is positive. The resulting effect is that the positive charge on the rod and the newly positive pithball will repel, and the pithball will swing *away from the rod.*

e.) You have a pithball that is covered with metallic paint. *Without allowing the pithball and rod to touch*, what clever thing could you do to make the pithball electrically negative?

Solution: Bring a positively charged rod close to the pithball. The valence electrons in the metallic covering will migrate toward the rod making the rod-side of the pithball negative and the off-side of the pithball electrically positive. If you touch the pithball on the off-side, you will "ground" that surface (i.e., electrons will flow from you to the exposed positive charge), neutralizing that side of the pithball. When the rod is then removed, the valence pithball's electrons will redistribute themselves over the pithball's metallic surface. But because there are now more electrons than before (remember, you transferred electrons from yourself to the pithball when you grounded the off-side), the surface will be electrically negative.

12.3) If you put gas in a spherical shell, the gas will distribute itself pretty much evenly throughout the volume. If you put charge on a solid metal sphere, what will the charge do?

Solution: Like charges repel. They try to get as far away from charges of their own kind as possible. When you put charge on a sphere, the charge distributes itself over the surface of the sphere attempting to get as far away from others of its kind as possible. As such, no charge will move inside the sphere--it will all be held in dynamic tension on the sphere's surface.



12.4) You have a charged, hollow, egg-shaped object made of copper. You put charge on the structure. How will the charge distribute itself over the surface? That is, will it distribute uniformly or what? If it doesn't distribute itself uniformly, how generally will it concentrate?

Solution: Due to the effect called *shielding*, charge densities go up on curved surfaces with the density getting larger and larger as the curvature gets more and more pointed. In other words, you will find more charge at the ends than in the middle, and more charge at the more pointed end than the less pointed end.

12.5) Two point charges, one twice as large as the other, are placed a distance r units apart. How will the force on the smaller charge change if:

a.) The distance is doubled?

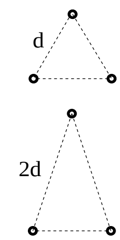
Solution: The Coulomb force is proportional to $1/r^2$. Double the distance and the force changes by $1/2^2$. In other words, it decreases by a factor of 4.

b.) The larger charge is doubled?

Solution: The Coulomb force is proportional to the size of each charge. Double one charge and the force will double.

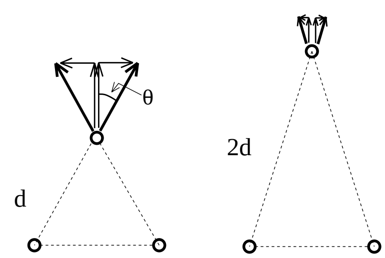
c.) How would the answers to *Parts a* and *b* have changed if you had been examining the larger charge instead of the smaller charge?

Solution: Coulomb force is a Newton's Third Law action/reaction pair (bad terminology, but you get the idea--for every force in the universe, there must exist an equal and opposite "reaction" force). In other words, the force the small charge experiences will be equal and opposite the force the large charge experiences.



12.6) Three equal point charges are positioned at the corners of an equilateral triangle. The net force on the top charge is measured. The distance between the top charge and the other two charges is doubled. Decide which of the lettered responses below describes how the new net force on the top charge will change, then explain why that response is appropriate.

a.) Double.
b.) Halve.
c.) Quadruple.
d.) Quarter.
e.) None of the above.



Solution: It turns out that *response e* is the correct one. The temptation is to figure that if you double the distance between the charges, the force will decrease by a factor of 4 (the force *is* proportional to $1/r^2$). In fact, that is true of the *magnitude* of the force. The problem is that force is a *vector*--you have to take direction into account. The net force is the sum of the *vertical components* of the forces on the top charge due to the presence of the other two charges. This number, in turn, is dependent upon the *angle* shown in the sketch. When you move the top charge, you change the angle and, hence, the vertical component of the acting forces. So although the force magnitudes go down by a factor of four, the net force is greater than expected because more of the forces are now in the vertical.

12.7) What does an *electric field* actually tell you? That is:

a.) Is it a vector? If so, what does its direction signify?

Solution: An electric field is a modified force field. Its direction at a particular point in space is defined as the direction a *positive charge* would accelerate if placed in the field at that point.

b.) What does its magnitude tell you?

Solution: The magnitude of an electric field function, as defined at a particular point in space, tells you the amount of *force per unit charge* that is available at that point due to the presence of the *field-producing* charge. That is, if you know the magnitude of E at a point, a charge q placed at that point will experience a force $F = qE$.

c.) How might electric fields be used in everyday life?

Solution: Electric fields motivate charge to move. Need light? Flipping a switch provides an electric field which moves the charge in the filament of your light bulb, and voila . . . light. Need toast? Flipping a switch provides an electric field which moves charge through the heating coils of your toaster. Need to frappé something? Flipping a switch provides an electric field that moves charge through a coil producing a magnetic field that is required if your blender's motor is to work. None of that would happen without an electric field to feed electrical energy into your system.

12.8) An electric field is oriented toward the right.

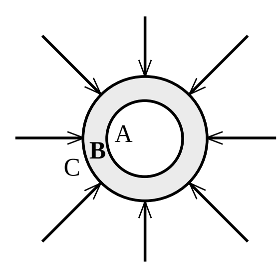
a.) What will an electron do if put in the field?

Solution: As positive charge moves *along* electric field lines (remember, the *direction* of an electric field line is defined as the direction a positive charge would accelerate if put in the field at the point of interest), a negative charge will move opposite the direction of the electric field lines.

b.) What will a proton do if put in the field at the same point as mentioned in *Part a*?

Solution: Protons and electrons have the same charge but different masses. When put in an electric field, the magnitude of the force each feels will be the same, but the directions will be opposite. Also, the accelerations will be different because the masses differ.

12.9) To the right is a cut-away cross-section of a thick-skinned ball. Given the electric field lines as shown:



a.) Tell me everything you know about *area A*. Note that you may not know *why* your observations make sense, but at least make them.

Solution: There are no electric field lines in *area A*, which is hollow, so there is no electric field in that region.

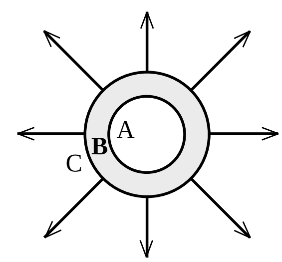
b.) Tell me everything you know about *area B*.

Solution: There are no electric field lines in *area B*, so there is no electric field there. I can't tell from the information given whether the shell is made up of a conducting material (i.e., a metallically bonded material) or an insulating material (i.e., a covalently bonded material).

c.) Tell me everything you know about *area C*.

Solution: Now it's getting interesting. The electric field lines in *area C* are pointed inward toward the sphere's surface. Electric field lines *leave* positively charged surfaces (this makes sense as a *positive test charge*--the kind of charge that is used to define the direction of an electric field--would be repulsed by a positively charged surface, hence the electric field lines would *leave* that surface) and *enter* negatively charged surfaces. As the field lines in this case are entering the surface, you can bet that the surface is negatively charged. Again, there is nothing to let us know whether the shell is a conductor or an insulator.

12.10) To the right is a cut-away cross-section of a thick-skinned ball. Given the electric field lines as shown:



a.) Tell me everything you know about *area A*. Note that you may not know *why* your observations make sense, but at least make them.

Solution: Again, there are no electric field lines in *area A*, which is hollow, so there is no electric field in that region.

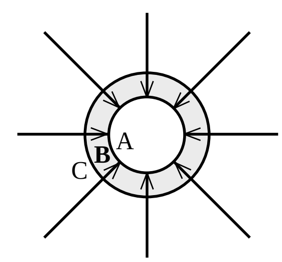
b.) Tell me everything you know about *area B*.

Solution: There are no electric field lines in *area B*, so there is no electric field there. I can't tell whether it is a conducting material or an insulating material.

c.) Tell me everything you know about *area C*.

Solution: In this case, the electric field lines are *leaving* the surface, so the surface must be positively charged.

12.11) To the right is a cut-away cross-section of a thick-skinned ball. Given the electric field lines as shown:



a.) Tell me everything you know about *area A*. Note that you may not know *why* your observations make sense, but at least make them.

Solution: There are no electric field lines inside the hollow, so there must be no electric field *and* no charge inside the hollow.

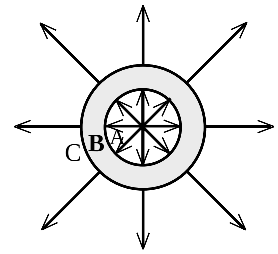
b.) Tell me everything you know about *area B*.

Solution: There is an electric field inside the solid, and it is oriented inward. That means two things. First, the solid must be an insulator, not a conductor. Second, there must be negative charge (remember, electric field lines *enter* negative charge) on the *inside surface* of the shell.

c.) Tell me everything you know about *area C*.

Solution: Because the electric field lines outside the shell appear to be mere extensions of the electric field lines inside the shell, it is probable that the electric field found outside the shell is produced by the negative charge on the inside surface of the shell.

12.12) To the right is a cut-away cross-section of a thick-skinned ball. Given the electric field lines as shown:



a.) Tell me everything you know about *area A*. Note that you may not know *why* your observations make sense, but at least make them.

Solution: There are outward directed field lines emanating from the center of the hollow, so there must be a positive charge Q at the center.

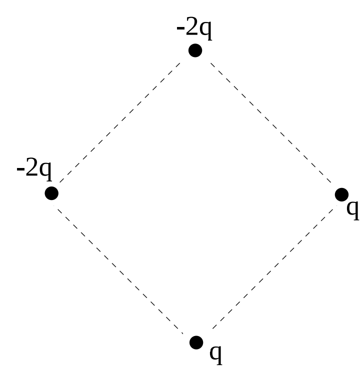
b.) Tell me everything you know about *area B*.

Solution: There are no electric field lines in the shell. If there had been no field lines *inside the hollow*, we would have no way of knowing whether the solid was a conductor or an insulator. Because there are field lines inside, we still don't know for sure if the shell is a conductor. How so? If the shell *was* a conductor, electrons in the shell would redistribute themselves so that *-Q's* worth of charge would spread over the shell's inside surface. The field produced by this negative charge would superimpose on the field produced by Q at the hollow's center, and the net effect would be zero electric field in the conductor. If the material was an insulator, on the other hand, *-Q's* worth of charge could have been placed on the shell's inside surface and the same net effect would be observed. In short, we can't tell what kind of material makes up the shell.

c.) Tell me everything you know about *area C*.

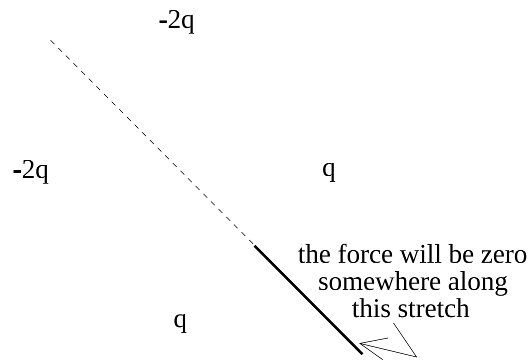
Solution: The field lines in this region mean there is an electric field. Where did it come from? It depends on whether the shell is an insulator or a conductor. If it is a conductor, the electrons that migrated to the inside surface of the shell would have left an equal amount of positive charge on the outside surface. That charge would produce an electric field comparable to the one shown. On the other hand, if the material was an insulator, then positive charge would have to have been placed on the outside surface. Either way, electric field lines exist outside the shell. That means there is an electric field in that region.

12.13) Consider the charge configuration shown to the right. You would like to place a negative charge in the field so that its acceleration is zero.



a.) Ignoring gravity, where might that be possible?

Solution: Due to symmetry, the position will be somewhere along the line shown in the secondary sketch. Assuming the charge was $-Q$, it would be repulsed by the *-2q's* and attracted to the *q's*. That eliminates the area inside the square where it will always be pulled upward and to the left. The area up and to the left is eliminated because the charge $-Q$ would always be closer to the larger negative charges and, hence, the smaller positive charges would never be able to counteract the *-2Q* effects. That means *-Q* would have to be somewhere along the line to the bottom right.



b.) Assuming you found a point that fits the bill (there may be more than one, but take just one), what do you know about the electric field at that point?

Solution: If the net force is zero at the point of interest, that would mean the *force per unit charge* available at that point would be zero. In other words, the electric field would have to be zero at that point.

12.14) What does an *absolute electrical potential* actually tell you? That is:

a.) Is it a vector? If so, what does its direction signify?

Solution: Electrical potentials are modified potential energy functions. As such, they are scalar fields and have no direction associated with them.

b.) What does its magnitude tell you?

Solution: If you know the absolute electrical potential at a particular point in an electric field, you know how much *potential energy per unit charge* is available at that point due to the presence of the field. That is, electrical potentials are modified potential energy functions. (Note that *electrical potentials* are often referred to simply as *potentials*.)

c.) How are electrical potentials used in everyday life?

Solution: *Electrical potential differences* and *electric fields* go hand in hand. That is, if you create an electrical potential difference between two points, you will also have created an electric field between those two points (and vice versa). That means that when you switch a lamp on in your home, the electric field that motivates current to flow through the lamp's light bulb is actually being produced by an artificially created *electrical potential difference* between the two prongs of the lamp's electrical cord.

12.15) An *electrical potential field* is oriented so that it becomes larger as you move to the right.

a.) What will a positive charge do if put in the field?

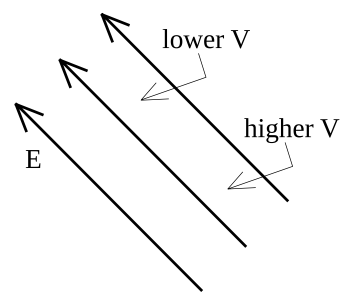
Solution: Positive charge "falls" through potential differences proceeding from *higher* to *lower* electrical potentials. In this case, that means they will accelerate to the left.

b.) What will a negative charge do if put in the field?

Solution: Negative charge does the opposite of positive charge, so they will accelerate from *lower* to *higher* electrical potential. In this case, that will be to the right.

c.) Is there an electric field associated with the potential and, if so, in what direction is it oriented?

Solution: By definition, the direction of an electric field is the direction a positive charge would accelerate if put in the field. As positive charge moves from higher to lower electrical potentials, electric field lines must be oriented from *higher* to *lower* electrical potentials.



12.16) A point charge exists at the origin of a coordinate axis. A distance 2 meters down the x axis, the electric field is observed to be 12 nt/C.

a.) What is the electrical potential at that point?

Solution: The *electric field* function for a point mass is kQ/r^2, whereas the *electrical potential* function for a point charge is kQ/r. As the r terms are the same in both cases (i.e., the distance between the field producing charge and the point of interest), the electrical potential expression is evidently just r times E in this case. That means the electrical potential is *(12 nt/C)(2 meters) = 24 volts*. (Note that a $nt\cdot m$ is an energy quantity, and energy divided by Coulombs is a *volt*.)

b.) You double the distance to 4 meters.

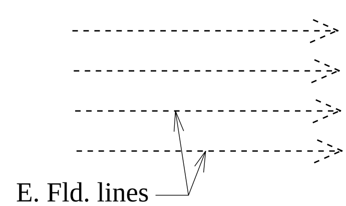
i.) What is the new electric field?

Solution: The electric field is a function of $1/r^2$. Doubling r will change the electric field by a factor of $1/(2)^2 = 1/4$.

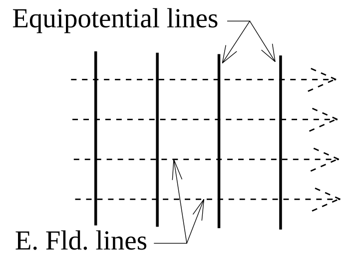
ii.) What is the new electrical potential?

Solution: The electrical potential is a function of $1/r$. Doubling r will change the electrical potential by a factor of $1/2$.

12.17) You have an electric field as shown. What will equipotential lines look like in the field?

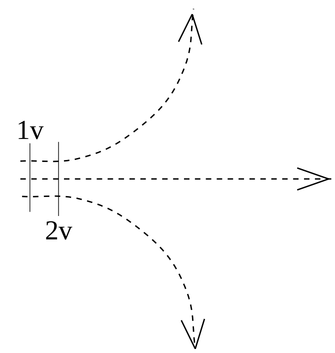


Solution: An equipotential line is a line upon which the electrical potential is the same at every point. It turns out that equipotential lines cut across electric field lines *at right angles*. So, for electric field lines to the right, the associated equipotential lines would be up and down (see sketch).



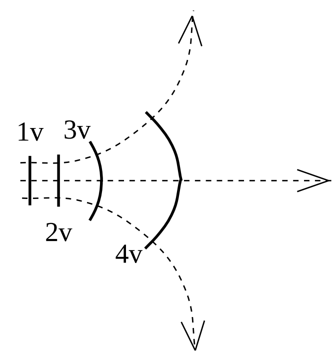
12.18) How is the *electrical potential difference* between two points related to the amount of work required to move a charge q from one point to the other?

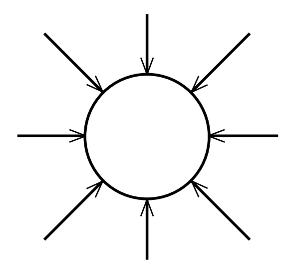
Solution: By definition, $W/q = -\Delta V$, so $W = -q\Delta V$.



12.19) The dotted lines in the sketch to the right are electric field lines. Also shown in the sketch are the *1 volt* and *2 volt* equipotential lines. Draw in the *3 volt* and *4 volt* equipotential lines.

Solution: What's tricky here is the fact that as the electric field weakens, the equipotential lines get farther apart. This follows both mathematically and logically. Mathematically, the relationship $\boldsymbol{E}\cdot\boldsymbol{d} = -\Delta V$ suggests that if $\boldsymbol{E}$ gets smaller (this is what happens as the electric field lines get farther apart), the distance $\boldsymbol{d}$ between incremental equipotential lines must get larger if the voltage change is to stay the same. From a common sense perspective, if voltage changes are related to the amount of work the electric field does as a charge goes from one point to another, then a lessened electric field intensity will do less work per unit displacement, and the net displacement will have to increase to effect the same amount of work. In any case, the equipotential lines will be perpendicular to the electric field lines, and farther apart as the size of the electric field diminishes.





12.20) To the right is a cut-away cross-section of a shell of radius a. Given the electric field lines as shown:

a.) What do you know about the electrical potential on the surface of the shell?

Solution: There are a number of bits and pieces of information you need to know to answer this question. To begin with, the electric field outside the shell is generated by a negative charge on the shell (look at the direction of the electric field lines), and there is no field inside the shell. Electrical potentials generated by negative charges are *negative*. In addition, if we assume the total charge on the shell is $-Q$, the magnitude of the electric field from the shell outward is kQ/r^2, where k is a constant and r is the distance from the shell's center to the point of interest. That is, assuming you deal with $r \geq a$, there is no difference between this electric field and the electric field you'd end up with if you got rid of the shell and replaced it with a point charge equal to $-Q$ located at the shell's center. Putting it in still another way, as viewed from outside the shell, the field-producing charge *looks* like a point charge. If you can't tell the difference between the two situations, the solution to one will fit the solution to the other. We know the electrical potential for a negative point charge. It is $k(-Q)/r$. This function also defines the electrical potential outside the shell . . . and upon its surface as well. Evaluating at $r = a$, we get $V = -kQ/a$ at the shell's surface.

b.) How would *Part a* have been different if the electric field lines had been oriented outward?

Solution: The electric field outside the shell would have to be generated by a net positive charge on the shell. Electrical potentials generated by positive charges are *positive*. On the shell the electrical potential would be kQ/a.

c.) What do you know about the electrical potential inside the cavity?

Solution: The *electric field* between two points is directly proportional to the *rate of change* of electric potential between the two points (that is, E is related to the spatial derivative of the electrical potential function or, in one dimension, dV/dx). As the electric field is zero inside the shell (no electric field lines), the *change* of the electrical potential between any two points inside the hollow will be zero--the electrical potential will be the same everywhere in that region. As electrical potentials are continuous, that value will also have to be the same as the electrical potential at the shell's surface, or $k(-Q)/a$.

d.) What do you know about the electric field at the boundary between the *inside* and *outside* of the shell?

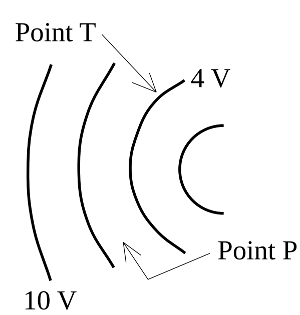
Solution: The electric field inside the shell is zero. Outside the shell, it is kQ/r^2. In other words, electric fields are *not* continuous functions.

e.) What do you know about the electrical potential at the boundary between the *inside* and *outside* of the shell?

Solution: As was said in *Part c*, the electrical potential at the boundary must be a continuous function.

f.) Where is the electrical potential zero?

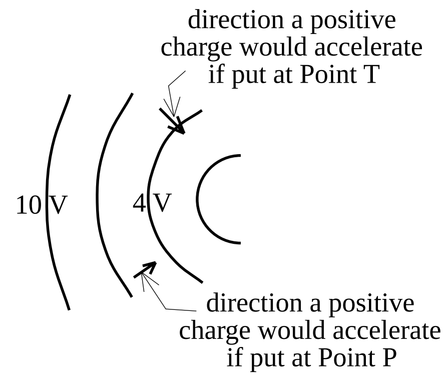
Solution: If you will remember back, in most cases a *potential energy* function is defined as zero where its associated force is zero. *Electrical potentials* are modified potential energy functions and have similar characteristics, relative to their associated *electric fields*. Although the electric field is zero for $r < a$, that is not due to a general dropping off of field intensity due to a moving away from the field producing charge. It is due to the fact that all the free charge carriers on the shell produce electric fields that add to zero inside the hollow. In short, for this configuration, we will take the electric field to be zero at infinity where the field has diminished due to distance, and we will take the electrical potential to be zero at that point also.



12.21) An oddly shaped charge configuration produces the equipotentials shown to the right.

a.) In what direction will a positive charge accelerate if placed in the field at *Point P*? How about *Point T*?

Solution: Positive charges follow electric field lines, and electric field lines are always perpendicular to equipotential lines (remember, an equipotential line is a line upon which the electrical potential is always the same). Because electric fields are oriented "downstream," so to speak, with respect to equipotentials, a positive charge will travel along a path that is perpendicular to the equipotential lines in its vicinity, and in such a way as to travel from *higher* to *lower* electrical potential. For ease of viewing, I've reproduced the drawing and drawn the solution for this question on that sketch.



b.) What would be different if a negative charge had been placed at *Point P*?
Solution: Negative charges do exactly the opposite of positive charges, so an electron would travel perpendicular to the equipotential lines and from *lower* to *higher* electrical potential.

c.) Is there any region in which:

i.) The magnitude of the electrical potential field is a constant? If so, identify it on the sketch.
Solution: That's what equipotential lines *are*--lines upon which the electrical potential is constant. There are *four* such lines depicted on our sketch, one for 10 volts, one for 7 volts, one for 4 volts, and one for 1 volt.

ii.) The magnitude of the electrical potential field is zero? If so, identify it on the sketch.
Solution: Given the equipotential lines, there must be a zero potential point just to the right of the far right line (i.e., the 1 volt line). (Note: voltages can be negative, so being zero doesn't mean you have necessarily hit the lowest potential possible.)